A STUDY OF THE SHORTEST DAY

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On a family walk in the Pentland Hills near Edinburgh just before Christmas in 2018, cousin Ian asked me about sundials. He said he had heard an item in the news about the shortest day. It was in December, but the latest sunrise would be in January. "Oh yes," I replied, "the media like to bring that one up every year. But you needn't worry, 21st December is the shortest day." Little did I realise that the news item was actually an interview with our BSS Chairman Frank King; sundial news travels far.[1]

The idea that sunrise keeps getting later, while daylight must be getting longer, is confusing. I first heard about it more than thirty years ago from a colleague at work. He used to say how much he liked having more daylight in January after the dark days of December. I think he learned about the times of sunrise from the information printed in his desk diary, which we all used to use at that time.

It seems to me I ought to get my story straight now. Someone else may want to ask about it. This study tries to put an explanation for sunrise and the shortest day into words and numbers for sundial people. Astronomers may approach their explanation in a different way.

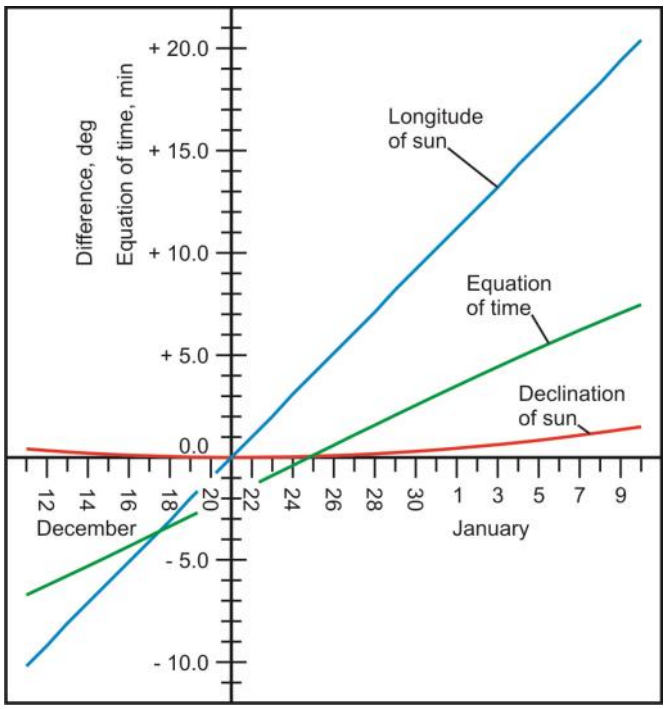

Fig. 1. Solar data over a period of 31 days. Longitude and declination of sun plotted as difference from respective values on 21 December. Equation of time plotted directly. Source of data: BSS Sundial Glossary.

Calculations

Aim and formula

The aim of this study is to calculate the time of sunrise and the length of day on the shortest day of the year, in the northern hemisphere, and to see what happens over a period of days before and afterwards. The basic calculation comes from the formula for sunrise, $\cos(h_{sr}) = -(-\tan\varphi \cdot \tan\delta)$ given in the *BSS Sundial Glossary* – where h_{sr} is the hour angle at sunrise (negative because it is before noon), φ is the angle of latitude, and δ is the declination of the sun.[2]

Even if you are uncomfortable with formulae, the ideas are understandable. Hour angle (h) is familiar from the equatorial sundial, where the hour lines are spaced apart by 15°. Latitude (φ) alters the time of sunrise, later in the day in winter in the north than in the south, for latitudes above the equator. Declination (δ) also affects the time of sunrise, which is latest in midwinter when the sun is lowest in the sky.

Now, sunrise is a number of hours before noon, sunset is the same number of hours after noon, and daylight is between sunrise and sunset, so length of day is the sum of the two. Note, the formula finds when the centre of the sun is on the horizon: it ignores refraction and twilight.

Data values

To use the formula and carry out calculations, data values are needed, which the *BSS Sundial Glossary* provides in convenient form. These are tables of values for declination of the sun, longitude of the sun, and equation of time, for 365 days of the year.[3] Transforming these values into a graph shows clearly how they vary over a period of days (Fig. 1). Note that 21 December has been used as a reference date, when the nominal value of sun declination is –23.4° and sun longitude is 270°. For the purposes of this study, and for clarity on the graph, sun declination and longitude have been plotted as differences from their respective BSS data on 21 December. The equation of time has been plotted directly.

The time period is 31 days from 11 December to 10 January. It shows a slow change in declination of the sun before and after its minimum midwinter value. The increase in longitude is almost linear day by day, reflecting Earth's orbit around the sun. The increase in equation of time is also almost linear over this period; it passes through zero on 25 December.

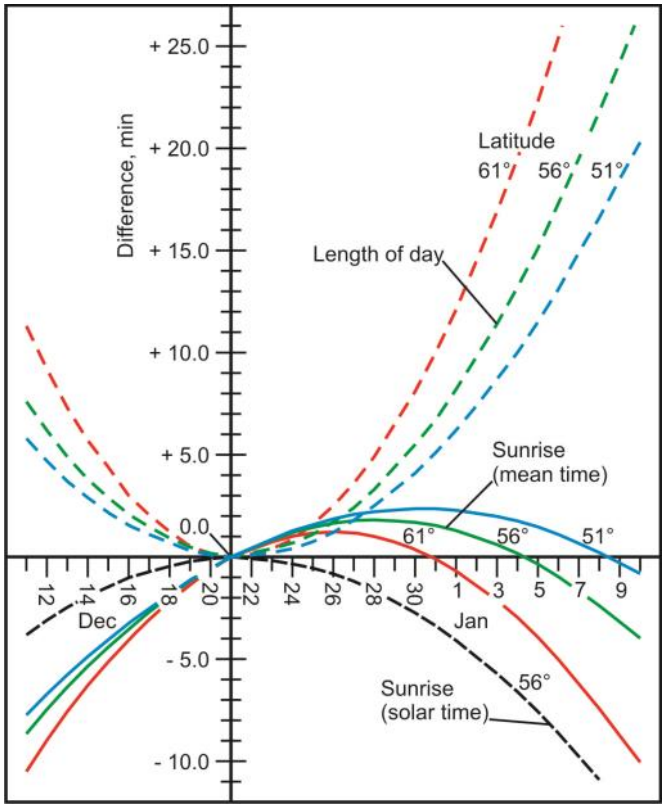

Fig. 2. Calculated values over a 31-day period. Time of sunrise (solar time), time of sunrise (mean time), and length of day, plotted as difference from respective values on 21 December. Selected latitudes 61°, 56°, and 51°. Source of basic data: BSS Sundial Glossary.

Solar time and mean time

Starting with calculations for latitude 56°, appropriate for the location of our family walk near Edinburgh, sunrise (solar time) is latest on 21 December as expected (Fig. 2). This is the shortest day. Before and after this date, sunrise is earlier. It is the broken line, coloured black. Like the plots for sun longitude and declination in Fig. 1, it is shown as a difference.

When solar time is adjusted to mean time with the Equation of Time applied, a new picture appears. The full line coloured green shows sunrise running later in mean time after 21 December, and this lateness continues until 4 January. The latest sunrise is on 28 December when it is just under 2 minutes later than it was on 21 December.

Note that length of day is not changed by the equation of time. The difference between sunrise and sunset is the same, whether measured in solar or mean time, and is shown as the broken line coloured green.

Effects of latitude

Plots for three different latitudes, 61°, 56°, and 51°, show a developing pattern. These latitudes span the British Isles from the north of Shetland to the south of England. At latitude 61° the sunrise curve rises and falls quicker than at 56°, and daylight progresses faster. At 51° it is the other way round: sunrise by the clock is later than it was on 21 December until 8 January.

Calculations for a wider range of selected latitudes are given in Table 1. Winter daylight is very short close to the Arctic Circle at latitude 66°, only 1h 45m. At latitude 66.6°, on the Arctic Circle, the shortest day has no daylight—the calculation formula fails at this point. Further south at 49.1°, there is a curious result that winter sunrise occurs at 8:00 am and winter daylight is 8 hours. This is the latitude of the most southerly point of the British Isles south of the Channel Islands. Continuing into lower latitudes, sunrise is earlier and daylight longer, until at the equator daylight is 12 hours.

Discussion and Summary

The shortest day and winter solstice

The shortest day is when declination of the sun is at a minimum. Strictly, winter solstice and minimum declination correspond to the point when the ecliptic longitude of the sun is 270°, which is a moment in time. The date of the shortest day is when this happens.

The date follows a leap year cycle. Winter solstice is about six hours later each year, which generally moves the date from 21 to 22 December in the fourth year. The leap year then re-adjusts the date. For example, winter solstice in 2018 was on 21st at 22:23 pm, this year 2019 it will be on 22nd at 4:19 am, and next year 2020 the leap year re-adjusts the date to 21 December at 10:02 am.

The shortest day is a convenient notion, but it is not a very exact event. For latitude 56°, the change in day length is less than 1 minute in 6 days from 18 to 24 December, which is not a big change. Winter solstice on the other hand is a precise moment in time.

Sunrise and sunset

Sunrise in the morning (mean time) gets later after the shortest day, then reaches a turning point and gets earlier again, as explained above and shown in Fig. 2. This is the

Latitude angle	Time of sunrise (solar time)	Hours of daylight
66°	11:07 am	1h 45m
61°	9:26 am	5h 8m
56°	8:40 am	6h 40m
51°	8:09 am	7h 42m
49.1°	8:00 am	8h 0m
40°	7:25 am	9h 9m
30°	6:58 am	10h 4m
20°	6:36 am	10h 47m
10°	6:18 am	11h 25m
0°	6:00 am	12h 0m

Table 1. Calculated time of sunrise and hours of daylight for 21 December at selected latitudes. Source of basic data: BSS Sundial Glossary.

Date in December	Sunrise (mean time)	Day length	Sunset (mean time)	Comment
14	8:33	6h 44m	15:17	Earliest sunset
21	8:38	6h 40m	15:18	Shortest day
28	8:40	6h 43m	15:23	Latest sunrise

Table 2. Summary of sunrise, day length, and sunset, on selected dates. Calculated for latitude 56°.

time of sunrise we see on the clock. Because of the effect of the EoT, it is quite correct to say that mornings continue getting darker after the shortest day. When sunrise passes its turning point, it is sensible to say that mornings have started getting lighter again. The date when sunrise returns to the time it was on the shortest day, which may be in January, is simply obtained by calculation.

Sunset in the evening (mean time) gets later when days are lengthening, as expected. But, in a mirror of what happens with sunrise, the time of sunset reaches a turning point *before* the shortest day. Again this is an effect of the EoT. In December, while days are still shortening, sunset gets earlier before its turning point and then becomes later, which continues through the shortest day and afterwards. It is therefore correct to say that evenings are becoming lighter before the shortest day.

Table 2 provides a summary of these curious sunrise and sunset effects. The numbers in the table reveal that both effects, sunrise getting later after the shortest day, and sunset getting later before it, are marginal. In the example given in the table for latitude 56°, the effects last for one week in each case. The time difference is only 2 minutes for sunrise, or 1 minute for sunset, but after two weeks on 28 December sunset is 6 minutes later, which may be noticeable.

Solar hours and mean hours

One final thought: solar hours are not precisely the same length as mean hours. Is it possible that, because of the EoT, day length alters as well as sunrise and sunset? Like many details of sundials, the answer is yes and no. There is a change but the difference is insignificant.[4]

Summary

The shortest day of the year is when the sun is lowest in the sky, as measured by the declination of the sun. In the northern hemisphere, the date is in midwinter on or around 21 December. On the shortest day sunrise is at its latest, in solar time. By the clock, in mean time, the latest sunrise is a few days afterwards, and sunrise may not return to its time on the shortest day until January. On the other hand, sunset begins to get later ahead of the shortest day, and evenings are lighter. These curious effects arise from the EoT, which changes quickly near the solstice. Day length and the times of sunrise and sunset also vary with latitude. This study has been interesting to carry out. Now that it is complete our family will have plenty to talk about, when we go out for our next winter walk.

ACKNOWLEDGEMENT

In his interview Frank King referred to the times of sunset before the shortest day. The discussion of sunset in this article was prompted by his broadcast, after listening to a replay on the BSS website.

REFERENCES and NOTES

1. Frank King: Radio interview, BBC Radio Scotland, broadcast on 21 December 2017, also on BSS website *http://sundialsoc.org.uk/news/winter-solstice/*
2. John Davis (ed.): *BSS Sundial Glossary: A source book of dialling data*, 2nd edn, British Sundial Society (2004).
3. The data in the tables are average values computed for the years 2000 to 2047, as explained in the Glossary.
4. The length of a solar day, from noon to noon, is variable while a mean solar day is 24 hours. When solar time runs slower than the clock, solar days are longer than 24 hours, and vice versa. The equation of time provides accurate figures for adjusting the time and is made up of daily increments. These increments are the actual amounts that the solar day is longer or shorter than 24 hours. The largest increment is +30 seconds in 1 day, that is 1¼ seconds in 1 hour. Therefore, the maximum length of 1 solar hour is 1h 0m 1¼s, instead of 1h 0m 0s, which is an insignificant difference.

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